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ON THE DECAY OF ANTIPROTONIC HELIUM ATOMS

S.I.Fedotov, O.I.Kartavtsev, D.E.Monakhov

The lifetime of long-lived antiprotonic helium atoms is mainly determined by the Auger decay. Rate of this process for a number of states of the $^3He\bar{p}e$ and $^4He\bar{p}e$ systems is calculated.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Распад антипротонных атомов гелия

С.И.Федотов, О.И.Картавцев, Д.Е.Монахов

Время жизни долгоживущих антипротонных атомов гелия определяется главным образом процессом оже-распада. Рассчитана скорость таких процессов для ряда состояний $^3He\bar{p}e$ и $^4He\bar{p}e$.

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1. Introduction

Metastable antiprotonic helium atoms $^{3,4}He\bar{p}e$ have been discovered in experiments on the delayed annihilation of antiprotons in helium media [1,2]. The observation of the resonant laser-induced annihilation [3,4] has initiated thorough investigations of these unusual systems. The discussion of modern theoretical studies of antiprotonic helium atoms and related topics can be found in [5].

The most important for clear understanding of an antiproton fate in the helium medium is the description of decay processes of antiprotonic helium atoms. In addition to the radiative transitions, the important decay channel in this system is the Auger decay, i.e., the emission of the electron with $^{3,4}He\bar{p}$ formation.

The main feature of the Auger decay rates of antiprotonic helium atoms is their essential dependence on the multipolarity, i.e., the angular momentum of the outgoing electron Δl. This feature was supposed already in [6] and justified in the early calculations of Russell [7]. Calculation of eigenenergies [8—10] unambiguously determines the smallest multipolarities of the Auger decay.

Up to now the only progress in the calculation of the Auger decay rates is due to the paper [11]. About three orders of magnitude decreasing of the Auger decay rates was found in these calculations with increasing the angular momentum of the outgoing electron from $\Delta l = 3$ to $\Delta l = 4$. Also the substantial decreasing of the Auger decay rates was found in

some cases due to the interference of different parts of the initial and final state wave functions.

In this note, much attention is paid to the calculation of the Auger decay rates in the important cases $\Delta l = 3, 4$ for a number of experimentally observed states of $^{3,4}He\bar{p}e$ systems. It is worthwhile to mention that up to now there are no other calculations of the decay rate for the $^{3}He\bar{p}e$ system.

2. Method

The Hamiltonian of the antiprotonic helium atom is

$$H = -\frac{1}{2\mu_3}\Delta\rho - \frac{1}{2\mu_{12}}\Delta\mathbf{r} + \frac{1}{|\rho - \beta_1 \mathbf{r}|} - \frac{2}{|\rho + \beta_2 \mathbf{r}|} - \frac{2}{r}, \tag{1}$$

where $\beta_1 = m_1/(m_1 + m_2)$, $\beta_2 = m_2/(m_1 + m_2)$; $1/\mu_{12} = 1/m_1 + 1/m_2$, $1/\mu_3 = 1/(m_1 + m_2) + 1/m_3$, $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, $\rho = r_3 - \frac{m_1\mathbf{r}_1 - m_2\mathbf{r}_2}{m_1 + m_2}$ are the Jacobi coordinates and m_1, $\mathbf{r}_1$, m_2, $\mathbf{r}_2$, m_3, $\mathbf{r}_3$ are masses and coordinates of the helium nucleus, antiproton and electron, respectively.

The notation l will be used for partial angular momentum of the pair of particles 1 and 2 and λ for the partial angular momentum of the third particle relative to the pair of particles 1 and 2.

The wave functions Ψ_{LN} and energy levels E_{LN} of the antiprotonic helium atom were obtained according to [10] as eigenfunctions and eigenvalues of the approximate hamiltonian $H_{LN} = P_{LN}HP_{LN}$, which explicitly is the projection of the hamiltonian H onto the closed channels subspace of the two-body $^{3,4}He\bar{p}$ system. In terms of the (l, λ) components of the wave functions P_{LN} is constructed as a projector onto the subspace defined by the condition $l > l_0$; l_0 is the largest pair angular momentum satisfying the condition $E^c_{l_0} < E_{LN}$, where $E^c_{l_0}$ is the energy of the two-body $^{3,4}He\bar{p}$ system of angular momentum l_0. Here and below the L, N notation of states, where L is the total angular momentum and N enumerates the states of the same L value, will be used.

The variational method was applied to solve the eigenvalue problem

$$(H_{LN} - E_{LN})\,\Psi_L = 0. \tag{2}$$

The variational trial functions $\varphi^{LM}_{inl\lambda}(\mathbf{r}, \rho)$ were chosen as product of the bispherical harmonics $\mathcal{Y}^{LM}_{l\lambda}(\hat{\mathbf{r}}, \hat{\rho})$ of angular variables $\hat{\mathbf{r}}$, $\hat{\rho}$ and simple radial functions $\varphi_{ipl}(r)$, $\varphi_{n\lambda}(\rho)$

$$\varphi^{LM}_{inl\lambda}(\mathbf{r}, \rho) = \mathcal{Y}^{LM}_{l\lambda}(\hat{\mathbf{r}}, \hat{\rho})\, \varphi_{il}(r)\, \varphi_{n\lambda}(\rho),$$

$$\varphi_{il}(r) = r^{l+p+1} e^{-a_i r},$$

$$\varphi_{n\lambda}(\rho) = \rho^{\lambda_n} e^{-b_n \rho}. \tag{3}$$

The continuous spectrum wave function Ψ_c will be orthogonal to the closed channels subspace and, therefore, orthogonal to Ψ_{LN}. The projector on the open channels subspace is denoted as $Q_{LN} = 1 - P_{LN}$ and λ_0 is denoted as the smallest possible angular momentum of the outgoing electron. Since the contribution to the Auger decay matrix element coming from the $(l, \lambda) \neq (l_0, \lambda_0)$ components of the continuous spectrum wave function Ψ_c is negligibly small, only the (l_0, λ_0) component will be taken into account.

Since the continuum wave function describes the electron scattering of large angular momentum, it can be taken as a product of the normalized hydrogen-like $^{3,4}\mathrm{He}\bar{p}$ wave function and function describing relative electron and $^{3,4}\mathrm{He}\bar{p}$ motion. For definite values of the total angular momentum and its projection this function is of the form

$$\Psi_c(\mathbf{r}, \rho) = \Phi(\mathbf{r}, \hat{\rho})\, f(\rho), \tag{4}$$

$$\Phi(\mathbf{r}, \hat{\rho}) = A(a, l_0)\, \mathcal{Y}^{L,M}_{l_0, \lambda_0}(\hat{\mathbf{r}}, \hat{\rho})\, r^{l_0} e^{-ar}, \tag{5}$$

where $A(a, l_0)$ is a normalization constant. In the framework of this approach the interaction of the outgoing electron and the remaining $^{3,4}\mathrm{He}\bar{p}$ hydrogen-like ion was described by the folding potential. The radial function $f(\rho)$ is the solution of the equation

$$\frac{1}{2\mu_3}\left(\frac{1}{\rho^2}\frac{\partial}{\partial\rho}\rho^2\frac{\partial}{\partial\rho} - \frac{\lambda_0(\lambda_0+1)}{\rho^2}\right) f(\rho) + (E - E^C_{l_0} - V_0(\rho))\, f(\rho) = 0, \tag{6}$$

satisfying the following boundary and asymptotic conditions

$$f(\rho) = \begin{cases} \rho^{\lambda_0} & \rho \to 0 \\ \dfrac{1}{\rho}\sin\left(\rho k - \dfrac{\lambda_0\pi}{2} - \eta \ln(2k\rho) + \delta\right), & \rho \to \infty. \end{cases} \tag{7}$$

Here $k = \sqrt{2\mu_3(E - E_{l_0}^C)}$ and phase shift δ is of no interest for this calculation. The folding potential $V_0(\rho)$ is defined by

$$V_0(\rho) = \langle \Phi | Q_{LN} H Q_{LN} | \Phi \rangle. \tag{8}$$

Due to the large centrifugal barrier for the outgoing electron, the transition matrix element is mainly determined by the wave function in the large ρ range. In this range with good accuracy the interaction of the electron and $^{3,4}He\bar{p}$ system is a sum of the Coulomb and centrifugal potential. Indeed, in our calculations the replacement of the folding potential by the Coulomb one gives rise to a minor change in the matrix element. This fact supports the applicability of the approximation used.

According to the Feshbach orthogonal projection method and having used the wave functions defined in (3), (4), (6), the decay rate λ is

$$\lambda = \frac{1}{\sqrt{2\mu_3(E - E_{l_0}^C)}} |M|^2 \frac{m_3 e^4}{\hbar^3} s^{-1}, \tag{9}$$

where the transition matrix element is

$$M = \langle \Psi_C | Q_{LN} H P_{LN} | \Psi_{LN} \rangle. \tag{10}$$

3. Numerical Results

Up to 600 trial functions were used in the calculation of eigenfunctions and eigenvalues for a number of the experimentally observed states of the $^{3,4}He\bar{p}e$ systems. Using these eigenfunctions in eq.(9), (10) the Auger decay rates λ have been calculated. These results

Table 1. Multipolarities Δl, energies E (a.u.) and Auger decay rates $\lambda(s^{-1})$ of $^4He\bar{p}e$

(L, N)	Δl	E	E [10]	λ	λ [11]
(35,4)	4	− 2.758086	− 2.759695	$\sim 5\cdot10^4$	$6.7\cdot10^2$
(35,4)	3	− 2.833794	− 2.835752	$7\cdot10^7$	$8.5\cdot10^7$
(34,3)	4	− 2.909770	− 2.910666	$\sim 1\cdot10^5$	$2.7\cdot10^4$
(33,3)	3	− 3.005968	− 3.007280	$1.2\cdot10^8$	$1.5\cdot10^8$

Table 2. Multipolarities Δl, energies E (a.u.) and Auger decay rates $\lambda(s^{-1})$ of $^3He\bar{p}e$

(L, N)	Δl	E	E [10]	λ
(34,4)	4	– 2.741186	– 2.744500	$\sim 10^5$
(33,4)	3	– 2.817797	– 2.821174	$2 \cdot 10^8$
(33,3)	4	– 2.894737	– 2.896675	$\sim 10^5$
(32,3)	3	– 2.993033	– 2.994695	$3.4 \cdot 10^8$

Table 3. Normalized contributions to the transition matrix element $M(l, \lambda)$ from the components of the wave function of partial angular momenta (l, λ) of the $^4He\bar{p}e$ system

$(L, N) = (34,3)$	$\Delta l = 4$	$(L, N) = (34,4)$	$\Delta l = 3$
(l, λ)	$M(l, \lambda)$	(l, λ)	$M(l, \lambda)$
(34.0)	– 0.28	(34.0)	0.24
(33.1)	0.51	(33.1)	– 0.46
(32,2)	– 0.38	(32.2)	1.22
(31.3)	1.14		

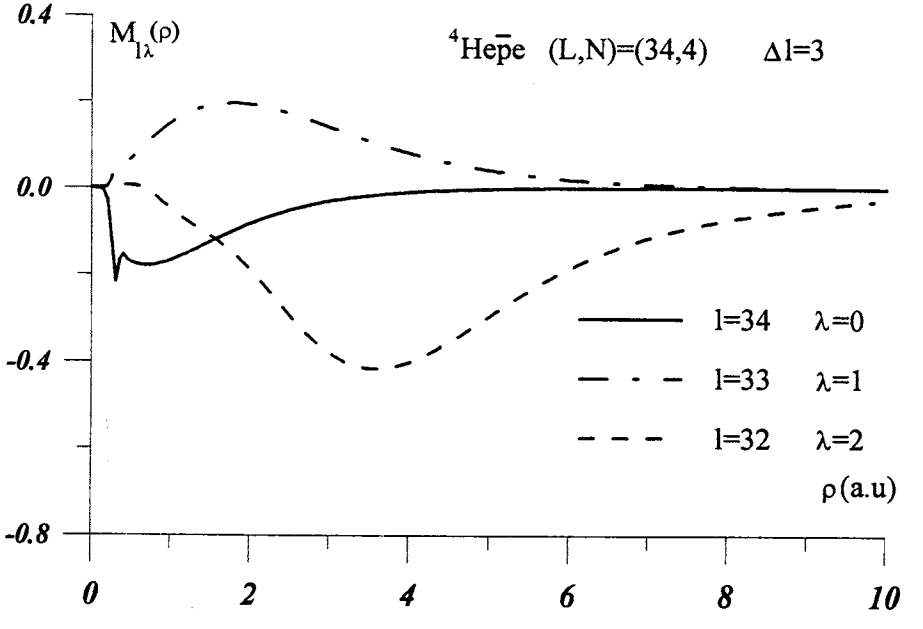

Fig. 1. Contribution to the matrix elements $M_{l\lambda}(\rho)$ (arbitrary units) from the (l, λ) components of the wave function as a function of the coordinate ρ

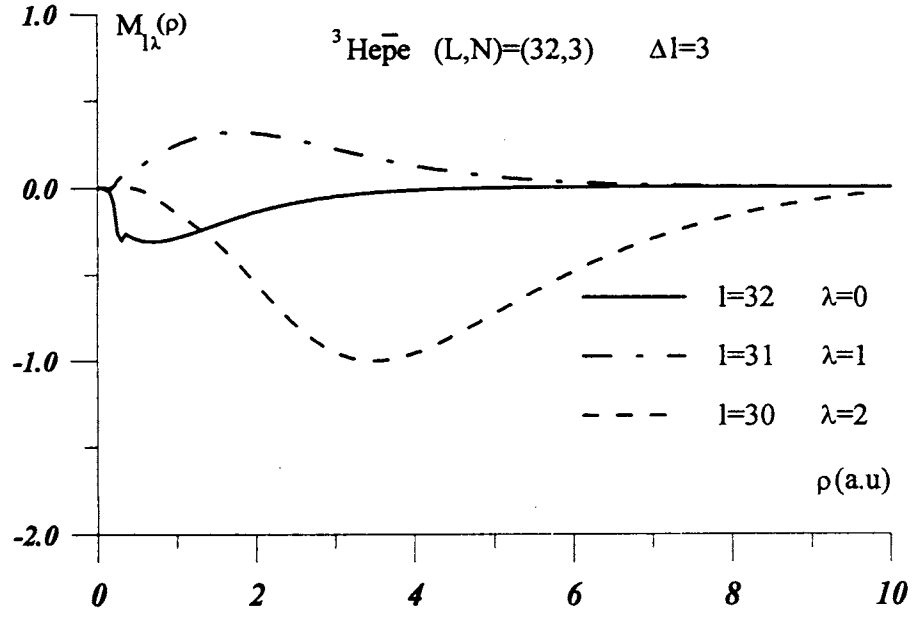

Fig.2. Contribution to the matrix elements $M_{l\lambda}(\rho)$ (arbitrary units) from the (l, λ) components of the wave function as a function of the coordinate ρ

are presented and compared with [10,11] in Tables 1 and 2. It is easily seen, that the usage of Jacobi coordinates in the present calculation instead of triangle coordinates in [10] provide the accurate enough energy values.

To understand the role of the wave function structure in this calculation, the largest contributions of the (l, λ) components to the transition matrix element are presented in Table 3. It is worthwile to mention that the behaviour of wave function at sufficiently large ρ values is important in evaluating the integral (10). By this reason, the ρ dependence of the largest (l, λ) component in the transition matrix element (10) is presented in Figs.1,2.

4. Discussion

As it follows from the numerical results, the essential point in the calculation of the Auger decay rates is to determine very fine features of the wave function. This is a rather complicated problem in the framework of the variational method. Really, the largest contribution to the matrix element comes from the smallest component (corresponding to the largest possible λ value) and those regions of the configuration space (large ρ), where these

components decrease exponentially. Apart from these difficulties, contributions of the (l, λ) components compensate each other due to the opposite signs.

As a consequence, uncertainty in the calculated decay rates for the multipolarity $\Delta l = 3$ is of an order of 10 per cent. As is indicated in Table 1, the decay rates for the $^4He\bar{p}e$ system in this case are in agreement with the results of [11]. The calculated decay rate of the (34,4) state of this system is in fairly good agreement with experimental lifetime 15 ns [3]. At the same time, for the (33,3) state the calculated decay rate twice exceeds the experimental value ($\tau = 16.6$ ns) [4].

On the contrary, the case of the transition multipolarity $\Delta l = 4$ is rather complicated for calculation and only an estimate of the decay rate can be obtained. These values exceed significantly the results obtained in [11].

The first calculation of the Auger decay rates of the $^3He\bar{p}e$ system shows the substantial isotopic dependence. In fact, decay rates of the (33,4) and (32,3) states of $^3He\bar{p}e$ are about three times larger in comparison with the corresponding (34,4) and (33,3) states of $^4He\bar{p}e$. At the same time, as it follows from the comparison of Fig.1 and Fig.2, the coordinate dependence in the transition matrix element (10) is quite similar for the $^4He\bar{p}e$ and $^4He\bar{p}e$ systems. It is worthwhile to mention, that other characteristics of the $(L-1, N)$ state of $^3He\bar{p}e$ are close enough in comparison with (L, N) state of $^4He\bar{p}e$. As for $\Delta l = 4$ transitions of $^3He\bar{p}e$, in this case the calculated decay rates can be estimated only up to an order of magnitude.

In conclusion, the calculation of the Auger decay rates is of great importance for understanding the antiprotonic helium. These values are defined by the small components of the wave function, and special methods will be developed to treat this problem with more accuracy.

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